

MATHEMATICS

1. $\sim p \wedge q$

2. p is false and q is true

3. $A^2 = I$
 $\therefore A^2 A^{-1} = I A^{-1}$
 $\therefore A = A^{-1}$

4. $[1 + 0 + 0 \quad 3 + 5x + 3 \quad 2 + x + 2] \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$

$$[1 \quad 5x + 6 \quad x + 4] \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$$

$$\therefore x + 5x + 6 - 2x - 8 = 0$$

$$\therefore 4x - 2 = 0$$

$$\therefore x = 2/4 = 1/2$$

5. We have $\sin 2A = \sin 2B + \sin 2C$

$$\therefore 2 \sin A \cos A = 2 \sin \left(\frac{2B+2C}{2} \right) \cos \left(\frac{2B-2C}{2} \right)$$

$$2 \sin (B + C) \cos (B - C)$$

$$2 \sin (180 - A) \cos (B - C)$$

$$2 \sin A \cos A = 2 \sin A \cos (B - C)$$

$$\cos A = \cos (B - C)$$

$$A = B - C$$

$$180 - (B + C) = B - C$$

$$\cos [180 - (B + C)] = \cos (B - C)$$

$$-\cos (B + C) - \cos (B - C) = 0$$

$$\cos (B + C) + \cos (B - C) = 0$$

$$2 \cos \left(\frac{B+C+B-C}{2} \right) \cos \left(\frac{B+C-B+C}{2} \right) = 0$$

$$2 \cos B \cos C = 0$$

$$\cos B \cos C = 0$$

$$\cos B = 0 \text{ OR } \cos C = 0$$

$$\therefore B = 90^\circ \text{ OR } C = 90^\circ$$

6. $7 \cos^2 \theta + 3 \sin^2 \theta = 4$
 $7 \cos^2 \theta + 3(1 - \cos^2 \theta) = 4$
 $7 \cos^2 \theta + 3 - 3 \cos^2 \theta = 4$
 $4 \cos^2 \theta + 3 = 4$
 $4 \cos^2 \theta + 3 = 1 + 3$
 $4 \cos^2 \theta = 1$
 $\cos^2 \theta = \frac{1}{4}$
 $\cos \theta = \pm \frac{1}{2}$
 $\cos \theta = \frac{1}{2} \quad \text{or} \quad \cos \theta = -\frac{1}{2}$
 $\cos \theta = \cos \frac{\pi}{3} \quad \cos \theta = \cos \frac{2\pi}{3}$
 $\therefore \theta = 2n\pi \pm \frac{\pi}{3} \quad \theta = 2n\pi \pm \frac{2\pi}{3}$

7. We have $(2 \cos x - 1)(3 + 2 \cos x) = 0$
 $2 \cos x - 1 = 0 \quad \text{or} \quad 3 + 2 \cos x = 0$
 $\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -\frac{3}{2}$
 $\cos x = -\frac{3}{2}$ which is not possible ($\because -1 \leq \cos x \leq 1$)

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$

8. 3

9. 1/5

10. parallel

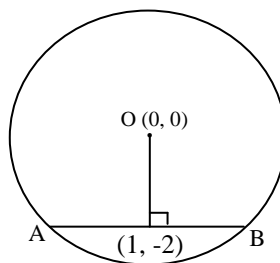
11. Slope of perpendicular from AB = $\frac{-2}{1}$

\therefore slope of AB = $\frac{1}{2}$

Eqⁿ of AB is $\frac{y+2}{x-1} = \frac{1}{2}$

$\therefore x - 1 = 2y + 4$

$\therefore x - 2y = 5$



12. 1.

Power of point (1, 2) w.r.t. circle $x^2 + y^2 - 5 = 0$ is $1 + 4 - 5$ which is 0. So, (1, 2) lies on the circle. Hence only one tangent can be drawn.

13. Centre of intersection of $2x + 3y + 4 = 0$ and $3x - y - 5 = 0$ which is $(1, -2)$.

Area is 154 sq. units.

\therefore so $r = 7$.

$$\text{Eq}^n (x - 1)^2 + (y + 2)^2 = 49$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 49$$

$$x^2 + y^2 - 2x + 4y - 44 = 0$$

14. Let $P(x, y)$ be a point on parabola, then

$$(x - 3)^2 + (y + 4)^2 = \frac{(x+y-2)^2}{2}$$

$$2(x^2 - 6x + 9 + y^2 + 8y + 16) = x^2 + y^2 + 4 + 2xy - 4x - 4y$$

$$\therefore 2x^2 - 12x + 18 + 2y^2 + 16y + 32 = x^2 + y^2 + 4 + 2xy - 4x - 4y$$

$$\therefore x^2 + y^2 - 2xy - 8x + 20y + 46 = 0$$

15. $\frac{2b^2}{a} = b \quad \therefore 2b = a$

$\therefore b/a = 1/2$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

16. $\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k} \quad \vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = (4)(-2) + (1)(3) + (-1)(-5)$$

$$= -8 + 3 + 5 = 0$$

$$\therefore (\vec{a} + \vec{b}) \perp (\vec{a} - \vec{b}) \quad \therefore \text{Angle is } \frac{\pi}{2}$$

17. Let given points be A, B, C, D & they are coplanar

$\therefore \vec{AB}, \vec{AC}, \vec{AD}$ are coplanar

$$\therefore \vec{AB} \cdot (\vec{AC} \times \vec{AD}) = 0$$

$$(\vec{b} - \vec{a}) \cdot [(\vec{c} - \vec{a}) \times (\vec{d} - \vec{a})] = 0$$

$$(\vec{b} - \vec{a}) \cdot [\vec{c} \times \vec{d} - \vec{c} \times \vec{a} - \vec{a} \times \vec{d} + \vec{a} \times \vec{a}] = 0$$

$$(\vec{b} - \vec{a}) \cdot [\vec{c} \times \vec{d} - \vec{c} \times \vec{a} - \vec{a} \times \vec{d} + 0] = 0$$

$$\vec{b} \cdot (\vec{c} \times \vec{d}) - \vec{b} \cdot (\vec{c} \times \vec{a}) - \vec{b} \cdot (\vec{a} \times \vec{d}) - \vec{a} \cdot (\vec{c} \times \vec{d}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{a} \cdot (\vec{a} \times \vec{d}) = 0$$

$$[\vec{b} \cdot \vec{c} \cdot \vec{d}] - [\vec{b} \cdot \vec{c} \cdot \vec{a}] - [\vec{b} \cdot \vec{a} \cdot \vec{d}] + [\vec{a} \cdot \vec{c} \cdot \vec{d}] + 0 + 0 = 0$$

$$[\vec{b} \cdot \vec{c} \cdot \vec{d}] - [\vec{a} \cdot \vec{b} \cdot \vec{c}] + [\vec{a} \cdot \vec{b} \cdot \vec{d}] + [\vec{c} \cdot \vec{a} \cdot \vec{d}] = 0$$

$$[\vec{b} \cdot \vec{c} \cdot \vec{d}] + [\vec{c} \cdot \vec{a} \cdot \vec{d}] - [\vec{a} \cdot \vec{b} \cdot \vec{d}] = [\vec{a} \cdot \vec{b} \cdot \vec{c}]$$

18. The actual d.c.'s of the lines are

$$\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7} \right) \text{ and } \left(\frac{3}{\sqrt{50}}, \frac{-4}{\sqrt{50}}, \frac{5}{\sqrt{50}} \right)$$

If θ is the angle between them, then

$$\begin{aligned} \cos \theta &= \pm \frac{(2)(3) + (3)(-4) + (-6)(5)}{7\sqrt{50}} = \pm \frac{6 - 12 - 30}{7\sqrt{50}} \\ &= \pm \frac{-36}{7\sqrt{50}} = \pm \frac{36}{7\sqrt{50}} \end{aligned}$$

19. The distance between the parallel planes $\vec{r} \cdot (2\hat{i} - \hat{j} + 3\hat{k}) = 4$ and

$$\vec{r} \cdot (2\hat{i} - \hat{j} + 3\hat{k}) = \frac{-13}{3} \text{ is}$$

$$\frac{4 - \left(\frac{-13}{3}\right)}{\sqrt{(2)^2 + (-1)^2 + (3)^2}} = \frac{\frac{25}{3}}{\sqrt{14}} = \frac{25}{3\sqrt{14}}$$

20. Since $\frac{1}{-2} = \frac{2}{-4} = \frac{3}{-6}$

i.e. drs of the lines are proportional

\therefore lines are parallel.

21. drs of the line joining (5, 2, 4) & (6, -1, 2) are -1, 3, 2

drs of the line joining (6, -1, 2) & (8, -7, k) are -2, 6, 2 - k

both lines will be collinear if their drs are proportional $\left(\frac{-1}{-2} = \frac{3}{6} = \frac{2}{2-k} \right)$

$$\therefore \frac{2}{2-k} = \frac{1}{2} \quad \therefore 2 - k = 4 \quad \therefore k = 2 - 4 = -2$$

22. The equation of the plane containing (1, 1, 0) is

$$a(x - 1) + b(y - 1) + c(z - 0) = 0 \quad \text{It also contains } (-1, 3, 4)$$

$$a(-1 - 1) + b(3 - 1) + c(4 - 0) = 0$$

$$\therefore -2a + 2b + 4c = 0$$

$$\therefore -a + b + 2c = 0$$

Also the plane is perpendicular to $x + y - 2z + 3 = 0$

\therefore By condition of perpendicularity, $a + b - 2c = 0$

$$\therefore \frac{a}{-4} = \frac{b}{0} = \frac{c}{-2} \quad \therefore a = 2, b = 0, c = 1$$

\therefore Required equation of plane $2(x - 1) + 0(y - 1) + 1(z - 0) = 0$

$$2x - 2 + 0 + z = 0 \quad \therefore 2x + z = 2$$

23. The equation of the plane passing through A(\vec{a}) and parallel to B(\vec{b}) and C(\vec{c}) is

$$(\vec{r} - \vec{a}) \cdot (\vec{b} - \vec{a}) \cdot (\vec{c} - \vec{a}) = 0$$

$$\text{Now } \vec{b} - \vec{a} = -2\hat{j} \quad \text{and} \quad \vec{c} - \vec{a} = -2\hat{i} - 4\hat{j} - 6\hat{k}$$

$$(\vec{r} - \vec{a}) \cdot (\vec{b} - \vec{a}) \cdot (\vec{c} - \vec{a}) = \begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 0 \\ -2 & -4 & -6 \end{vmatrix} = 0$$

$$\therefore (x-1)(12-0) - (y-1)(0-0) + (z-1)(0-4) = 0$$

$$\therefore 12x - 12 - 0 - 4z + 4 = 0$$

$$\therefore 12x - 4z - 8 = 0$$

$$\therefore 3x - z - 2 = 0$$

24. The minimum value of z at 0 is 0.

25. OABC is the feasible region

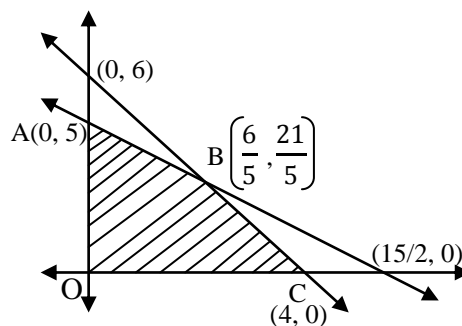
$$Z(O) = 15(0) + 10(0) = 0$$

$$Z(A) = 15(0) + 10(5) = 50$$

$$Z(B) = 15(6/5) + 10(21/5) = 18 + 42 = 60$$

$$Z(C) = 15(4) + 10(0) = 60$$

\therefore Maximum value of Z is 60.



26. $u = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right), \quad v = \sin^{-1}(3x-4x^3)$

$$x = \sin \theta, \quad x = \sin \theta$$

$$u = \tan^{-1}\left(\frac{\sin \theta}{\cos \theta}\right), \quad v = \sin^{-1}(\sin 3\theta)$$

$$u = \theta = \sin^{-1} x \quad v = 3\theta = 3\sin^{-1} x$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}, \quad \frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{3}$$

27. As required circle touches y-axis at the origin.

∴ Let Centre of the circle is d (a, 0) and radius is a

∴ Equation of circle will be,

$$(x-a)^2 + (y-0)^2 = a^2$$

$$\Rightarrow x^2 - 2ax + a^2 + y^2 = a^2$$

$$\Rightarrow x^2 + y^2 - 2ax = 0 \quad \dots(i)$$

By differentiating above equation w.r.t. x, we get

$$2x + 2y \frac{dy}{dx} - 2a = 0$$

$$\therefore 2a = 2x + 2y \frac{dy}{dx} \quad \dots(ii)$$

From (i) and (ii),

$$x^2 + y^2 - \left(2x + 2y \frac{dy}{dx}\right)x = 0$$

$$\Rightarrow x^2 + y^2 - 2x^2 - 2xy \frac{dy}{dx} = 0$$

$$\Rightarrow x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

28. Given, $f(x) = e^x (\sin x - \cos x)$

$$\therefore f'(x) = e^x [\cos x + \sin x] + [\sin x - \cos x]e^x$$

$$\therefore f'(x) = 2e^x \sin x$$

To verify Rolle's Theorem.

$$f'(c) = 0$$

$$2e^c \sin c = 0$$

$$\Rightarrow \sin c = 0 \quad \therefore c = \pi$$

29. Let $I = \int \frac{dx}{\sqrt{8+2x-x^2}}$

$$\Rightarrow I = \int \frac{dx}{9+2x-x^2-1}$$

$$\Rightarrow I = \int \frac{dx}{3^2 - (x-1)^2}$$

$$\therefore I = \sin^{-1} \left(\frac{x-1}{3} \right) + c$$

30. As given,

$$f(x) = \begin{cases} \frac{1}{5}, & 0 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Now, probability of waiting time not more than 4 is $= 4 \times \frac{1}{5} = 0.8$

31.
$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$$

$$\therefore \lim_{x \rightarrow 0} x \sin \frac{1}{x} = k$$

$$\Rightarrow 0 = k$$

32. As given,
$$f(x) = \begin{cases} \frac{\log(1+2x) \sin x^\circ}{x^2} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$$

Is continuous at $x = 0$

$$\therefore \lim_{x \rightarrow 0} \frac{\log(1+2x) \cdot \sin x^\circ}{x^2} = k$$

$$\therefore \lim_{x \rightarrow 0} \frac{2 \cdot \log(1+2x)}{2x} \cdot \lim_{x \rightarrow 0} \frac{\sin \left(x \times \frac{\pi}{180} \right)}{\left(x \times \frac{\pi}{180} \right)} \times \frac{\pi}{180} = k$$

$$\Rightarrow 2 \times \frac{\pi}{180} = k$$

$$\Rightarrow \frac{\pi}{90} = k$$

33. By using anti differentiation method,

We will get to know that, Option (C) is correct.

i.e.
$$\frac{a^{x+\tan^{-1}x}}{\log a} + c$$

34. Degree $\Rightarrow 3$ Order $\Rightarrow 2$

35. $A = \int_0^2 (2x - x^2) dx$

$$\Rightarrow A = \left[x^2 - \frac{x^3}{3} \right]_0^2$$

$$\Rightarrow A = \left[4 - \frac{8}{3} \right]$$

$$\Rightarrow A = \frac{4}{3} \text{ sq units.}$$

36. Given $\int \frac{f(x)}{\log(\sin x)} dx = \log [\log \sin x] + C$

By using anti differentiation method, we will get

$$\frac{d}{dx} (\log [\log \sin x] + c)$$

$$= \frac{1}{\log(\sin x)} \cdot \frac{1}{\sin x} \cdot \cos x$$

$$= \frac{\cot x}{\log(\sin x)} \quad \therefore f(x) = \cot x$$

37. Let $I = \int_0^{\pi/2} \frac{\sqrt[n]{\sec x}}{\sqrt[n]{\sec x} + \sqrt[n]{\cos ec x}} dx \quad \dots(i)$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt[n]{\sec\left(\frac{\pi}{2} - x\right)}}{\sqrt[n]{\sec\left(\frac{\pi}{2} - x\right)} + \sqrt[n]{\cos ec\left(\frac{\pi}{2} - x\right)}} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt[n]{\cos ec x}}{\sqrt[n]{\cos ec x} + \sqrt[n]{\sec x}} dx \quad \dots(ii)$$

Adding equation (i) and (ii)

$$2I = \int_0^{\pi/2} dx$$

$$\Rightarrow 2I = [x]_0^{\pi/2} \quad \Rightarrow I = \frac{\pi}{4}$$

38. $y(1 + \log x) \frac{dx}{dy} - x \log x = 0$

$$\Rightarrow \left(\frac{1 + \log x}{x \log x} \right) dx = \frac{dy}{y}$$

Integrating on both side

$$\int \left(\frac{1 + \log x}{x \log x} \right) dx = \int \frac{dy}{y}$$

$$\log(x \log x) = \log y + \log C$$

$$\Rightarrow \log(x \log x) = \log(y \cdot c)$$

$$\therefore x \log x = y \cdot c \quad \dots(i)$$

As $x = e, \quad y = e^2$

$$\therefore e = e^2 \cdot c \quad \therefore c = \frac{1}{e}$$

Putting $c = \frac{1}{e}$ in eq (i) we get

$$x \log x = \frac{y}{e} \quad \Rightarrow \quad y = ex \log x$$

39. Given. I.F of $\frac{dy}{dx} + py = Q$ is $\sin x$

$$\therefore e^{\int P dx} = \sin x$$

$$\Rightarrow \int P dx = \ln(\sin x)$$

By anti-differentiation method, we will get

$$P = \frac{d}{dx} [\ln(\sin x)] = \frac{1}{\sin x} \cdot \cos x$$

$$\therefore P = \cot x$$

40. Given $n = 5, \quad p = \frac{1}{3}$

$$\therefore q = \frac{2}{3}$$

$$p(2 < x < 4) = p(x = 3)$$

$$= {}^5C_3 \left(\frac{1}{3} \right)^3 \left(\frac{2}{3} \right)^2$$

$$= \frac{5 \times 4}{2} \times \frac{1}{27} \times \frac{4}{9} = \frac{40}{243}$$

41. $\Sigma P(x) = 1$

$$P(1) + P(2) + P(3) + P(4) = 1$$

$$30k = 1$$

$$\therefore K = \frac{1}{30}$$

42.
$$P\left(\frac{1}{4} < x < \frac{1}{3}\right) = \int_{1/4}^{1/3} f(x) dx$$

$$= \int_{1/4}^{1/3} 3(1 - 2x^2) dx$$

$$= [3 - 2x^3]_{1/4}^{1/3}$$

$$= \frac{179}{864} = 0.2071$$

43. P (more heads than tails)

$$= P(x = 5 \text{ or } 6 \text{ or } 7 \text{ or } 8)$$

$$= P(x = 5) + P(6) + P(7) + P(8)$$

$$= \frac{1}{28} \left[8C_3 + 8C_2 + 8C_1 + 8C_0 \right]$$

$$= \frac{93}{256}$$

44. $\frac{dy}{dx} = \log(x + 3)$ i.e. $dy = \log(x + 3) dx$

Integrating

$$y = x \log(x + 3) - \int \frac{x}{x+3} dx + C$$

$$y = (x + 3) \log(x + 3) - x + C$$

solution is $y = (x + 3) \log(x + 3) - x + 1$

45. $\sqrt{1 + x^2} + \sqrt{1 + y^2} = a(x - y)$

Diff w.r.t 'x'

$$\frac{dy}{dx} = \sqrt{\frac{1 + y^2}{1 + x^2}} \cdot \frac{a\sqrt{1 + x^2} - x}{a\sqrt{1 + y^2} + y}$$

But from the given relation

$$\begin{aligned}
 \left[\sqrt{1+x^2} + \sqrt{1+y^2} \right] &= \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\
 &= a(x-y) \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\
 &= 1+x^2 - 1 - y^2 = a(x-y) \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\
 &= x+y = a \left[\sqrt{1+x^2} - \sqrt{1+y^2} \right] \\
 &= y + a\sqrt{1+y^2} = a\sqrt{1+x^2} - x \\
 &= \frac{a\sqrt{1+x^2} - x}{a\sqrt{1+y^2} + y} = 1 \\
 \therefore \frac{dy}{dx} &= \sqrt{\frac{1+y^2}{1+x^2}}
 \end{aligned}$$

Clearly it is a differential equation of first order and first degree.

46. The area bounded by the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ is given by
- $$\begin{aligned}
 &\frac{a^2}{4} (\pi - 2) \text{ sq. units} \\
 &= \frac{36}{4} (\pi - 2) \text{ sq. units} \\
 &= 9(\pi - 2) \text{ sq. units}
 \end{aligned}$$

47. By area of the region rounded by $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16 ab}{3}$

48.
$$I = \int_0^{\frac{\pi}{2}} \frac{1}{3 \sin \theta + \sqrt{g - g \sin^2 \theta}} \cdot 3 \cos \theta \, d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\cos \theta}{\sin \theta + \cos \theta} + \frac{\sin \theta}{\cos \theta + \sin \theta} \right) d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} d\theta \quad \therefore I = \frac{\pi}{4}$$

49. $\int \frac{(x^2-1) dx}{(x^2+2x+1)x\sqrt{x+\frac{1}{x}+1}}$ is equal to

$$\int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(1 - \frac{1}{x} + 2\right)\sqrt{x + \frac{1}{x} + 1}}$$

$$\int \frac{2z dz}{(z^2+1)z}$$

Putting $x + 1/x + 1 = z^2$

$$= \left(1 - \frac{1}{x^2}\right) dx = 2z dz$$

$$= 2 \int \frac{dz}{1+z^2}$$

$$= 2 \tan^{-1} z + C$$

$$= 2 \tan^{-1} \left(\sqrt{\frac{x^2+x+1}{x}} \right) + C$$

50. $x^2 + y^2 = 5$

$$2x + 2y \frac{dy}{dx} = 0$$

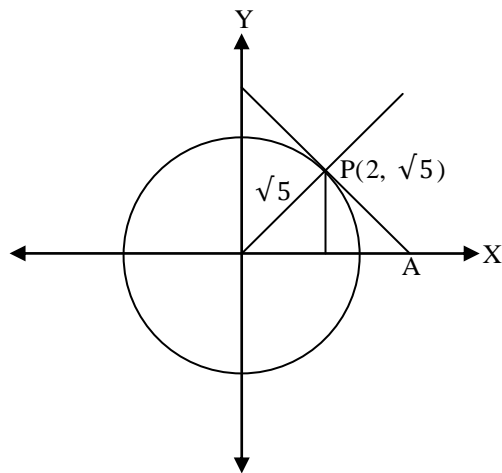
$$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{5}}$$

The equation of tangent is

$$(y - \sqrt{5}) = \frac{-2}{\sqrt{5}}(x - 2)$$

The equation of normal is

$$(y - 5) = \frac{\sqrt{5}}{2}(x - 2)$$



The tangent and normal intersect at x-axis at the points $A(9/2, 0)$ and $O(0, 0)$ respectively.

$$\therefore A (\Delta OAP) = \frac{1}{2} \times \frac{9}{2} \times \sqrt{5} = \frac{9\sqrt{5}}{4}$$